

## VII-6. A LIGHT BEAM WAVEGUIDE USING HYPERBOLIC-TYPE GAS LENS

Yasuharu Suematsu, Kenichi Iga, and Shinichi Ito

*Tokyo Institute of Technology, Tokyo, Japan*

This paper is concerned with the optimum design of a light beam waveguide constructed with the proposed lens-like media which have hyperbolic temperature distribution. In such media, the temperature distribution is ideally quadratic in the transverse direction, and so mode conversion loss originating from higher-order variation of the dielectric constant is small. Moreover, this guide has the merits that the design procedure is clear, the mode matching at the input port is easy, and it can be constructed using ordinary air. This consideration can easily be extended to a guide with a curved configuration. It is shown that the experimental convergency of this gas lens is in agreement with theoretical considerations.

Hyperbolic-Type Gas Lens. The transverse cross-section of the proposed hyperbolic-type gas lens is shown in Figure 1. Two pairs of hyperbolic heat-conductive pipes are heated or cooled by  $+\Delta T$  or  $-\Delta T^0$  C compared with the temperature  $T_0$  on the center axis, and it is assumed that there is no convection.

In this case, the dielectric constant  $\epsilon$  of the medium is calculated as follows, under the condition that  $T_0 \gg \Delta T (x^2 - y^2)/a^2$ :

$$\epsilon = \epsilon(0) \left[ 1 - (gx)^2 + (gy)^2 \right] ,$$
$$g = \sqrt{(\epsilon_r - 1) \Delta T / (\epsilon_r T_0) / a} , \quad \epsilon(0) = \epsilon_0 \epsilon_r \quad (1)$$

where  $\epsilon_0$  and  $\epsilon_r$  are the dielectric constants of a vacuum, and the specific dielectric constant of the gas at  $T_0$ , respectively. This medium has a convergent property in the transverse x direction, and a divergent property in the y direction.<sup>1</sup>

The heating power  $w$  per unit axial length is given by:

$$w = 4 \kappa \Delta T \quad (\text{watts/meter}) \quad (2)$$

where  $\kappa$  is the specific heat conductivity of the medium.

Optimum Design of the Beam Waveguide. The proposed beam waveguide is constructed with hyperbolic-type gas lenses of axial length  $l_g$  as shown in Figure 2. The spacing is  $l_0$ , and each neighboring gas lens is mutually rotated around the axis by  $90^\circ$ .

The natural mode of the waveguide is an axially symmetric Hermite-Gaussian beam, and the spot size of the guided beam becomes maximum at the center part of the convergent section ( $= s_c$ ), and minimum at the center part of the divergent section ( $= s_d$ ) in the x and y directions, respectively. The maximum spot size is:<sup>1</sup>

$$\frac{s_c}{w_0} = \left[ \cot^2 \varphi \cdot \frac{\tan \varphi + \tanh \varphi + Q}{-\cot \varphi + \coth \varphi + Q} \cdot \frac{\tan \varphi + \coth \varphi + Q}{\cot \varphi - \tanh \varphi - Q} \right]^{1/4} \quad (3)$$

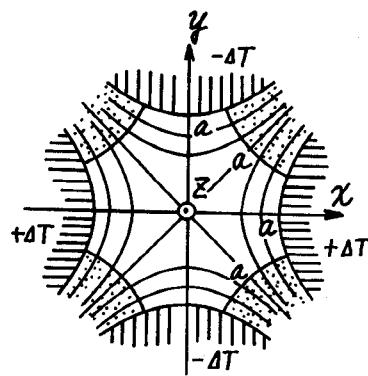


Fig. 1. Hyperbolic Temperature Distribution of Gas Lens.  
Dotted and hatched areas are thermal insulators and conductors, respectively.

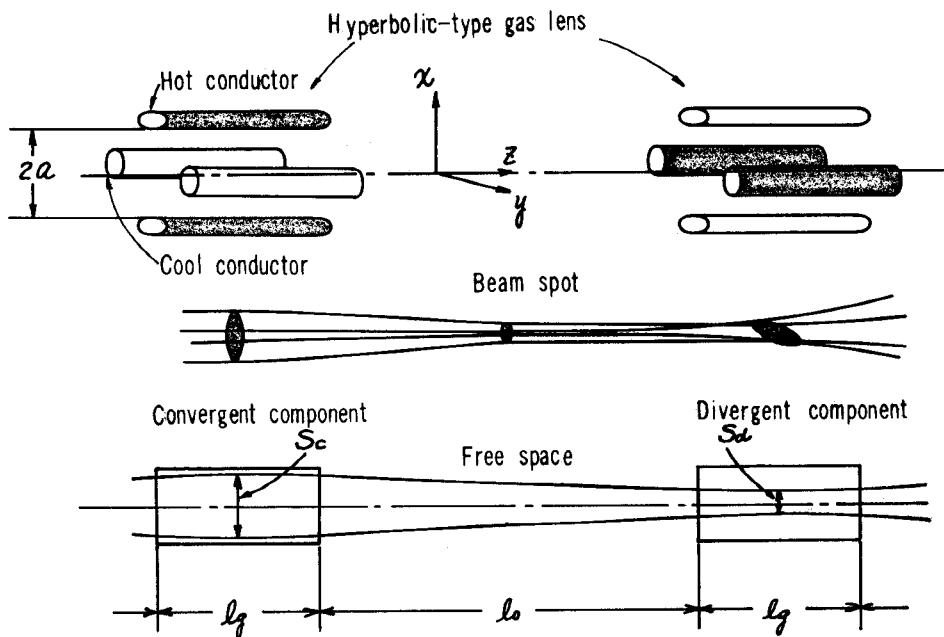


Fig. 2. Beam Waveguide Using Hyperbolic-Type Gas Lenses,  
periodically rotated 90° around center axis.

where  $\varphi = \frac{1}{2} g l_g$ ,  $w_0 = 1/\sqrt{k(0)g}$ ,  $k(0) = \omega \sqrt{\epsilon(0)\mu}$  and  $Q = g l_0 = 2\varphi(l_0/l_g)$ .

The beam stability is maintained in some regions of  $\varphi$  and it is given by the minimum value  $(s_c/w_0)_m$  at  $\varphi = \varphi_m$ , and these results are shown in Figure 3. To determine  $a$ , the parameter

$$(s_c/a)_m = (s_c/w_0)_m \cdot (w_0/a) \quad (4)$$

is introduced and decided by the diffraction loss. The parameters, for example, are as follows: when  $\lambda = 0.63 \mu$  and  $\Delta T = 2.5^\circ C$  with ordinary air (at 1 atm,  $20^\circ C$ ), then  $a = 5 \text{ mm}$ ,  $s_c = 1 \text{ mm}$ ,  $l_g = 2.7 \text{ m}$ , and  $l_0 = 0$ .

Diffraction and Mode Conversion Losses. The guided light beam is interrupted by the walls of the medium and the accompanying loss is considered to be a diffraction loss,  $\eta_d$ . It is approximately calculated as follows:

$$\eta_d = \frac{4}{\sqrt{2\pi}} \int_{\sqrt{2}X_1}^{\infty} e^{-t^2/2} dt, \quad (5)$$

where  $X_1 = a/s_c$ . The parameter  $(s_c/a)_m$  in (4) is determined by considering the value of  $\eta_d$ . Moreover, the mode conversion loss originating from non-quadratic distribution of the dielectric constant of the gas lens must be taken into account. In this gas lens,  $10^{-5}$  of the energy is converted to the higher modes per km because of the fourth-order variation of the dielectric constant when  $l_0 = 100 \text{ m}$ .

Mode Matching. One has to match the incident axial symmetric mode to the natural mode of the system. This mode matching is performed with two or three cylindrical lenses. The method of mode matching can be formulated using matrices.<sup>1,2</sup>

Experimental Convergency of the Hyperbolic-Type Gas Lens. The experimental gas lenses are shown in Table I and Figures 4a and 4b. Brass cylinders were used instead of ideal pipes with hyperbolic cross-sections.

The temperature of the cool pair of pipes was maintained at room temperature with radiators, and the hot pair was heated by the heater. The gas used was ordinary air (1 atm,  $20^\circ C$ ). Thermocouple probes were used for measuring the wall temperature. The temperature distribution was controlled to be symmetric between the paired pipes and flat in the axial direction. In the case of gas lens C in Table I, which had rather large dimensions, a thermistor probe was used to determine the temperature distribution in the gas lens medium, which was quadratic as in eq. (1) under working conditions of the lens. The arrangement of Fig. 4b was superior to that of Fig. 4a for producing a symmetrical temperature distribution.

To confirm the design theory, we made some experiments on the convergency of hyperbolic-type gas lenses. For types A and B in Table I,  $TEM_{00}$ -mode Gaussian beam ( $\lambda = 0.63 \mu$ ) was used. The output beam through the experimental gas lens was deformed to an asymmetric pattern as in Figs. 4d and 4e, for which spot sizes in the x and y directions were  $w_x$  and  $w_y$ , respectively. Measured ratios of the spot dimensions vs. distance from the gas lens are shown in Figure 5, with corresponding theoretical values. The measured results were in good agreement

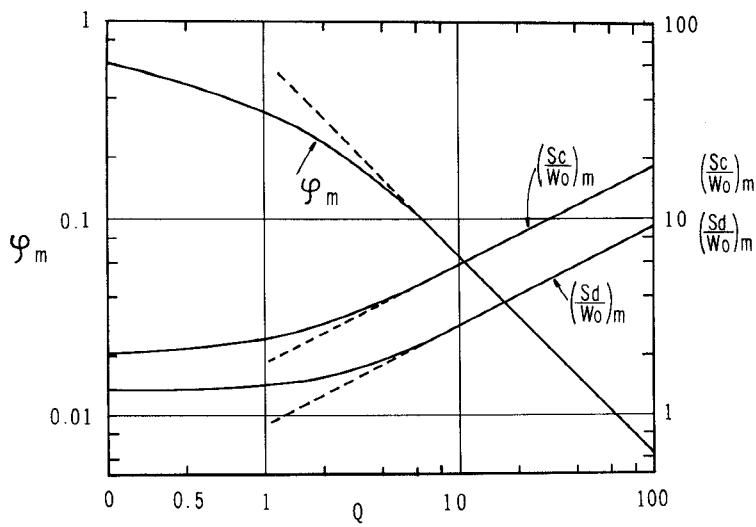


Fig. 3.  $\varphi_m$ ,  $(s_c/w_0)_m$  and  $(s_d/w_0)_m$  vs.  $Q = g l_0$  under the optimum condition.

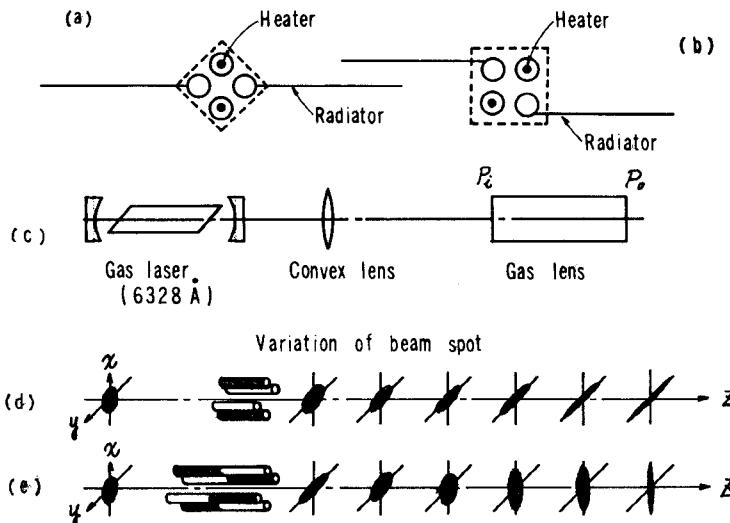


Fig. 4. Experimental Arrangement for Measurement of Convergence of Hyperbolic - Type Gas Lens.

TABLE I

| Gas Lens | $l_g$ (m) | $2a$ (mm) | Type      | Section |
|----------|-----------|-----------|-----------|---------|
| A        | 1.0       | 10        | Fig. 4(a) | half    |
| B        | 2.1       | 10        | Fig. 4(b) | unit    |
| C        | 2.1       | 38        | Fig. 4(b) | half    |

with the theoretical ones. For gas lenses A and B with  $2a = 10$  mm, the effects of convection and gravity could not be detected for temperature differences  $2\Delta T$  less than approximately  $10^\circ C$ . But for gas lens C, with  $2a = 38$  mm, these effects were remarkable when  $2\Delta T$  was over  $1.7^\circ C$ . Since the deformation of the laser beam was not detectable under this condition, an auto-collimator was used for measuring the focal length.

The focal length of gas lenses is expressed approximately as  $f = 1/(g^2 L_g)$  when  $g$  in eq. (1) is very small compared with unity. The measured focal length was over 200 m and in agreement with this theoretical value when  $2\Delta T$  was less than  $1.7^\circ C$ .

#### References.

1. Y. Suematsu and H. Fukinuki, "Analysis of the Idealized Light Waveguide Using Gas Lens", J. IECE of Japan, vol. 48, pp 1684-1690, October 1965.
2. H. Kogelnik, "Matching of Optical Modes", Bell Syst. Tech. Jrnl., vol. 43, pp 334-337, January 1964.

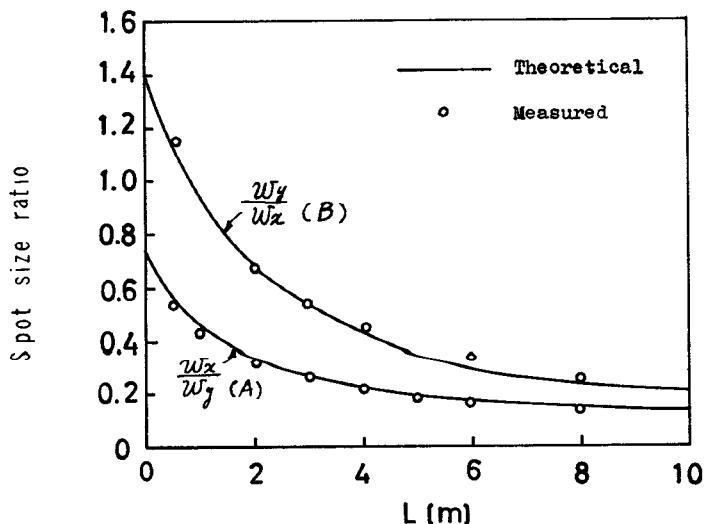


Fig. 5. Experimental Results on Spot Size Ratios vs. Distance from output end of Gas Lens. In Case A, measured values correspond to response of half-section gas lens A as shown in Fig. 4(d). In Case B, values correspond to response of unit section gas lens B as shown in Fig. 4(e).

**MICROWAVE DEVELOPMENT LABORATORIES, INC.**  
87 Crescent Road, Needham Heights, Mass 02194

Design, Development and Production of Microwave Components and  
Sub-assemblies from 400 Mc to 70 Gc.

**ADTEC (ADVANCED TECHNOLOGY CORP.)**  
1830 York Road, Timonium, Maryland

Millimeter and Submillimeter Wave Receivers, Radiometers,  
Detectors, Mixers, Harmonic Mixers, Harmonic Generators,  
Modulators, Calibration Loads.